Investment Decisions and Trading Emissions Permits under Uncertainty

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Abstract

In this paper we analyse a polluting firm which selects an investment portfolio to maximize expected profit under an environmental regulation in the presence of trading emissions permits market. Trading emissions permits would be expected to become one of the useful methods in achieving the environmental goals in an efficient way. Our concern is to consider an interaction between the abatement investment and the usage of trading emissions permits to investigate how polluting firms strategically react to the environmental policy.

The main results we obtained is that, with emissions permits trading, penalty cost can not affect the firm’s abatement investment decisions. Only the price of emissions permits can affect such decisions. So, how the emissions permits market works (or how the price are determined in the market) is one of the most important problems to consider.

KEYWORDS: uncertainty, trading emissions permits, abatement investment

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I. Introduction

To support sustainable development, and to control greenhouse gas (GHG) emissions, Kyoto Protocol in 1997 COP3 had adopted Kyoto Mechanism, which includes three methods: trading emissions permits (TEPs), joint implementation (JI), and clean development mechanism (CDM). These methods are intended to use market mechanisms and effective control. Especially, because TEPs has economic theoretical foundations, it has been paid attention not to practitioner but economic perspective. In fact, EPA (Economic Planning Authority) has already implemented and developed Sulfur Dioxide (SO2) emissions trading market in US. Regarding to GHG, UK domestic emissions trading scheme started in 2003. Also, EU will begin in 2005, and TEPs among Annex I Parties in Kyoto Protocol will in 2008. In Japan, Ministry of the Environment has trial to domestic TEPs scheme.

Then, we analyse a polluting firm’s decision-making problems on an environmental abatement investment when she can trade GHG emissions permits in an inter-firm TEPs market. TEPs are expected to become one of the powerful methods in achieving the environmental goals. However, several previous papers have shown that stochastic permit prices may reduce firm’s incentive to invest in abatement capitals or technologies. It is important to encourage the development and market penetration of advanced pollution abatement technology. Hence, we add to the previous research by focusing on the interaction between an abatement investment and an usage of TEPs to investigate strategic behavior of polluting firm’s investment decision towards the environmental policy.

The main results of this paper are as follows. First, we show that the levels of abatement investment depend only on the permits prices regardless of costs of penalties, which the government imposes on polluting firms. This result may imply that environmental policies such as mandatory instruments (e.g. penalties) may not work deliberately in the presence of TEPs. Second, we observe that the market structure of TEPs and its price would play important role of firm’s investment decisions.

There are strands of literature related to this research. One is the literature on the effects of TEPs on polluting firms’ reactions in the market. Malleg (1989) argues that marketable permits may not give greater incentives than standards regulation, because the incentive effects of marketable permits depend on whether polluting firms are buyers or sellers. Laffont and Tirole (1996) analyzed the impacts of spot and futures markets for TEPs on the polluting firms, and showed that stand-alone spot markets induce excessive investment. Xepapadeas (1999) analyses the reaction of polluting firm’s regarding expansion of abatement investment and location decision, in the presence of environmental policy that takes the form of emission taxes or tradable permits. Zhao (2003) develop a rational expectations general equilibrium model of permits trading and irreversible abatement investment. Zhao (2003) shows that tradable permits might help maintain firms’ investment incentive under various uncertainty conditions.

The remainder of this paper is organized as follows. In section 2 we introduce our model. A representative firm’s investment decision making is discussed in section 3. In section 4, we describe an interpretation of the results in section3. Conclusions and comments on further reach are in section 5.
II. Model

In this section, we describe assumptions and notations used throughout this paper.

Suppose that \((\Omega, \mathcal{F}, P)\) is a probability space, and that \(W = (W_1, W_2, W_3)\) is a 3-dimensional Brownian motion process. \(\{\mathcal{F}_t\}_{t \geq 0}\) denotes the standard Brownian filtration.

Fix \(T > 0\), and we call \(T\) a closing date. Define a stochastic process \(X_1\) as follows: for \(\forall t \geq 0\),
\[
X_1(t) = \mu_1(t) + \sigma_1 W_1(t)
\] (1)

where \(\mu_1 : [0, \infty) \rightarrow (0, \infty)\) is a twice continuously differentiable increasing function with \(\mu_1'' < 0\), and \(\sigma_1\) is a positive constant. We shall put an interpretation on \(X_1(T)\). A firm invests \(x\) to its business at time 0, and the investment has strong influence on its profit. However, it also depends on uncertainty, e.g. consumer behavior, competitors, materials prices, and so many business conditions. Therefore, we use \(X_1(T)\) as the operating profit during \([0, T]\).

Let us define an \(\mathcal{F}_T\)-measurable random variable
\[
X_2 = e^{\mu_2(x) + \sigma_2 \rho W_1(T) + \sqrt{1 - \rho^2} W_2(T)}
\] (2)

where \(\mu_2 : [0, \infty) \rightarrow (0, \infty)\) is a twice continuously differentiable increasing function such that \(e^{\mu_2(x)} \{ (\mu_2')^2(x) + \mu_2''(x) \} > 0\), \(\sigma_2\) is a positive constant, and \(\rho < 1\). Clearly there is relevance between the production, the profit, and the quantity of GHG which the firm discharges. While the amount of investment of the firm is a decisive factor common to them, randomness may affect the emission. Accordingly \(X_2\) is applied to express GHG emissions produced over the accounting period by the business with initial investment \(x\).

Let \(b : [0, \infty) \rightarrow [0, \infty)\) be a twice continuously differentiable, increasing, and strictly concave function with \(b(0) = 0\). For any \(y \in [0, \infty)\), \(b(y)\) is used to refer to decrease in GHG emissions when the firm makes an emission reductions investment \(y\) at time 0. The examples are renewable energy projects, energy improvement projects, switching fossil fuels, emission reductions in the physical distribution system, and so on. Then \(X_2 - b(y)\) represents net emissions of GHG.

A positive constant \(K\) is called capital which the firm borrowed from a bank at time 0 for the operation in \([0, T]\).

\(c, \alpha, \rho\) are positive constants. In our model, \(c\) is called a GHG emission limitation on the firm over the period, and if the net emissions of GHG is greater than \(c\), the firm must pay the penalty \(\alpha\) per one unit of excess GHG, i.e. the total amount of the penalty is \(\alpha (X_2 - b(y) - c)\). \(\rho\) is interpreted as the emissions trading price per one unit of GHG at time 0. Additionally, a positive constant \(r\) is called continuous compounding interest rate.

III. Investment decision-making

In this section, we shall solve following three investment problems. To begin with, we will show a way of dealing with an investment problem, where the firm does not use GHG emissions trading. This is a benchmark for the other problem. Then, we shall provide an answer to a problem for the firm which takes GHG emissions trading positions only at the opening date.
Investment decision-making without GHG emissions trading

Our problem is

\[
\begin{align*}
\text{maximize} & \quad E \left[ X_1(T) - K(e^{rT} - 1) - \alpha \cdot \max(X_2 - (b(y) + c), 0) \right], \\
\text{subject to} & \quad K = x + y, \; x \geq 0, \; y \geq 0.
\end{align*}
\]

We will interpret this maximizing problem. At time 0, the firm borrows an amount of money $K$, and allocate the funds between the business $x$ and the emission reduction project $y$. The operating profit will fluctuate stochastically during the accounting interval $[0, T]$. At time $T$, the firm must pay interest expenses $K(e^{rT} - 1)$, and GHG emission penalty $\alpha(X_2 - (b(y) + c))$ if net GHG emissions $X_2 - b(y)$ would exceed the limit $c$. Thus, the firm chooses a portfolio $(x, y)$ to maximize the expected net income earned during the accounting period $[0, T]$.

We shall now prescribe the solution. Let us compute the right-hand side of (3), and substitute $y = K - x$. This yields

\[
J_1(x) := E \left[ \right. X_1(T) - K(e^{rT} - 1) - \alpha \max(X_2 - (b(K - x) + c), 0) \left. \right]
= \mu_1(x)T - K(e^{rT} - 1) - \alpha \cdot g(x)\Phi(d_1(x, K - x, 0))
+ \alpha(b(K - x) + c)\Phi(d_1(x, K - x, 0) - \sigma_2\sqrt{T})
\]

where

\[
g(x) = e^{\mu_2(x) + \frac{1}{2}\sigma_2^2 T},
\]

\[
\Phi(u) = \int_{-\infty}^{u} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}v^2} dv,
\]

\[
d_1(x, y, z) = \begin{cases} 
\log \frac{g(x)}{b(y) + \frac{K-x-y}{p} + z + c} + \frac{1}{2} \sigma_2^2 T & \text{for } b(y) + \frac{K-x-y}{p} + z + c > 0 \\
\infty & \text{otherwise}
\end{cases}
\]

Therefore we have following three cases on the problem.

If we have

\[
\frac{dJ_1}{dx}(0) > 0 \quad \text{and} \quad \frac{dJ_1}{dx}(K) < 0,
\]

there exists an unique $x^* \in (0, K)$ such that

\[
\frac{dJ_1}{dx}(x^*) = 0,
\]

and $x^*$ maximizes $J_1$ uniquely, i.e. $(x, y) = (x^*, K - x^*)$ is the unique maximizer of (3).

In case that either

\[
\frac{dJ_1}{dx}(0) \leq 0, \quad \text{or} \quad \frac{dJ_1}{dx}(K) \geq 0
\]

holds \textsuperscript{1}, $(0, K)$ or $(K, 0)$ maximizes (3) respectively.

\textsuperscript{1}Both inequalities do not hold simultaneously.
(ii) Investment decision-making with GHG emissions trading

Here, we shall solve

\[
\begin{align*}
\text{maximize} & \quad E \left[ X_1(T) - K(e^{rT} - 1) - \alpha \cdot \max(X_2 - (b(y) + z + c), 0) \right], \\
\text{subject to} & \quad K = x + y + pz, \ x \geq 0, \ y \geq 0.
\end{align*}
\]

From the viewpoint of economics, this problem is interpreted as follows. At time 0, the firm tries to make a decision to maximize the expected net income earned during \([0, T]\). According to the purpose, she selects a portfolio composed of the business \(x\), the emission reduction project \(y\), and tradable GHG emission permits \(z\) units. In this problem, the penalty is expressed by \(\alpha \cdot \max(X_2 - (b(y) + z + c), 0)\).

Then, we will solve the second problem. By calculating the expectation in (5), and substituting \(z = \frac{K - x - y}{p}\), we obtain

\[
\begin{align*}
J_2(x, y) & := E \left[ X_1(T) - K(e^{rT} - 1) - \alpha \cdot \max \left( X_2 - \left( b(y) + \frac{K - x - y}{p} + c \right), 0 \right) \right] \\
& = \mu_1(x)T - K(e^{rT} - 1) - \alpha \cdot g(x)\Phi(d_1(x, y, 0)) \\
& \quad + \alpha \left( b(y) + \frac{K - x - y}{p} + c \right) \Phi(d_1(x, y, 0) - \sigma_2\sqrt{T}).
\end{align*}
\]

Furthermore, by the assumption on \(b\), if there exists \(\hat{y} \geq 0\) such that

\[
\frac{\partial^2 J_2}{\partial x^2}(x, \hat{y}) = 0,
\]

it must be unique. If \(\frac{\partial J_2}{\partial y}(y) < \frac{1}{p}\) for any \(y \geq 0\), we set \(\hat{y} = 0\).

There may be the following cases about the second problem.

If there exists \(\hat{x} \geq 0\) satisfying

\[
\frac{\partial J_2}{\partial x}(x, \hat{y}) = 0,
\]

\(J_2\) is uniquely maximized for \((\hat{x}, \hat{y})\), i.e. \((x, y, z) = (\hat{x}, \hat{y}, \frac{K - \hat{x} - \hat{y}}{p})\) is the unique solution of (5).

Suppose that

\[
\frac{\partial J_2}{\partial x}(x, \hat{y}) < 0 \quad \text{for } \forall x \geq 0,
\]

then \((0, \hat{y})\) is the unique maximizer of (5).

If either

\[
\frac{\partial J_2}{\partial y}(y) > \frac{1}{p} \quad \text{for } \forall y \geq 0, \quad \text{or} \quad \frac{\partial J_2}{\partial x}(0, \hat{y}) > 0 \quad \text{for } \forall x \geq 0
\]

holds, \(J_2\) fails to attain a maximum.
IV. Economic Implications

These analysis we conducted are quite simple, and might seem to be unusual setting of economic paper. However these results have some important economic implications.

In the previous section, we consider a representative firm’s investment decision problems in two cases, that is with-without emissions trading. In either case, the firm would have to pay penalty in closing period $T$ if she would not be able to attain her emission levels previously agreed. While the former case, we can interpret, the firm is not allowed to utilize GHG emissions permits at all, or the market does not exist, in the latter case, the firm can trade them.

In the former case, without trading emissions permits, we showed that there exists the unique solution $(x^*, y^*)$ for the firm’s maximizing expected net income(profit) problem (3). To maximize expected return, the firm could select the business investment amount $x^*$ and the emissions abatement investment $y^*$ at time 0. Also, in the latter case, with emissions trading, there exists the unique solution $(\tilde{x}, \tilde{y}, \tilde{z})$ for the firm’s expected net income(profit) maximization problem (5). In that case, the firm will choose optimal $x$, $y$, and $z$, where $z$ is interpreted as quantity of tradable emissions permits.

We did not show explicitly in this paper, but following result had also obtained. That is, compared with-without emissions permits, there are some cases in which

$$x^* < \tilde{x}.$$  

Here we note the interpretations on this result. The inequality means that the business investment level with trading emission permits is higher than the level without trading. Such result suggests that trading emissions permits would make more investment opportunities for profit. Therefore it might be attained more social efficient outcome with-trading case than without.

However, the main and important result we obtained is equation (7) in the previous section. Equation (7) is one of the conditions for the firm’s expected net income maximization problem with emissions trading, i.e. (5), and $\beta'(\tilde{y}) = \frac{1}{p}$. This equation (7) can be interpreted as follows. As mentioned above, $b$ is a function that is interpreted as levels of reduction by investment $y$, and an increasing function of $y$. Then, the derivative $b'$ represents marginal abatement, and (7) means that the marginal abatement is equal to the reciprocal of $p$, where $p$ is GHG trading price per unit at time 0.

In this model, if a firm would not be able to attain her emissions levels which ex ante agreed at the closing date $T$, she would have to pay penalty related to $\alpha$ of unit penalty cost. Therefore there seems to be that $\alpha$ affects the firm’s abatement investment decision, but can not affect at all. Wherever levels of $\alpha$ would be set, the firm does not consider it when she decide to the emission reduction project $y$. Only emissions permits price $p$ at time 0 can affect the firm’s decision about abatement investment. Then, in our settings, it means that public authority(government) can not make firms to have some incentive for abatement investment by penalty. This outcome may have important implication when we argue the mechanism design of inter-firm emissions trading market.

In sum, we obtain following observation:

Observation 1: Under the conditions we set in the previous section, in case with emissions trading, levels of investment for abatement depend only on the GHG emissions trading price per unit at time 0 regardless of other parameter such as levels of penalty.
Note that this observation depends on the setting such that the firm can take any short positions of emissions permits \( z \). If short selling is regulated or restricted, these outcomes would differ.

Next, we examine market price of GHG emissions permits. In our analysis, unit price of GHG emissions trading \( p \) is exogenously given. So, we did not consider and interpret how the price is determined in the market. In fact, such market price would depend on market structure of trading GHG emissions permits.

Additionally, considering the market structure of emissions trading and its prices, we obtain following implications. We set two cases to explain. The first is the case where emissions trading price \( p_1 \) is set in a competitive market, while the second case is that a price \( p_2 \) is not set in a competitive market. In the latter case, we express the situation that there are a few trader firms with market power. Then, compared to two cases, the second case price \( p_2 \) would be expected higher than the competitive market price \( p_1 \). That is \( p_2 > p_1 \) because of market power of some traders. Sometimes \( p_1 \) is called welfare maximizing price level and \( p_2 \) is called distortional price level in trading market of GHG emissions permits.

From the assumptions, \( b'(y) \) is a strictly decreasing function in \( y \). So other things being equal, if \( p \) increases, then \( \frac{1}{p} \) decreases, and \( \hat{y} \) such that \( \frac{1}{p} = b'(\hat{y}) \) increases. In short, \( \hat{y} \) is an increasing function of \( p \) in (7). Then, for \( 0 < p_1 < p_2 \),

\[
\hat{y}(p_1) < \hat{y}(p_2).
\]

That is, if trading market of GHG emissions is not competitive and there are upper price distortion in it, each firm (which is price taker and can not affect prices) with trading, has incentive to over-investment for abatement \( y \). Such investment level might bring economic inefficient outcome.

In sum:

**Observation 2**: If there are some distortion in trading market of GHG emissions permits, and the market price is higher than competitive one, then a representative firm would have more investment for abatement than competitive GHS emissions trading market and such outcome might bring some welfare loss.

We focus only on a representative firm’s investment problem, but these results bring important economic problems, which are due to inefficiency of emissions trading markets. These interpretation seem to be important for designing of such trading GHG market, or comparing cost-effectiveness to other economic instruments such as taxes to abate GHG.
V. Conclusion and further research

In this short paper, we analyzed a representative firm’s investment problems under uncertainty, and interpreted some characteristics of solutions in the case of emissions permits trading. The main results we obtained is summarized in observation 1 and observation 2 in the previous section. That is, with emissions permits trading, penalty cost can not affect firm’s abatement investment decisions. The market price of trading of emissions permits can only affect such decision. Therefore, what makes structure of emissions permits market and how price are determined is important problem to consider.

To further research, we have to address three points.

To begin with, we do not study any cases in a polluting firm could buy and sell GHG emissions permits during the accounting period $[0, T]$. Especially, we explain a case which can be regarded as significant also from a practical viewpoint.

For $\forall t \geq 0$, $\mathcal{G}_t := \sigma(\{W_1(s)\}_{0 \leq s \leq t})$, $X_3(t) := E(X_2 | \mathcal{G}_t) = e^{\mu t} + \frac{1}{2}\sigma^2 t + \frac{1}{2}\sigma^2 \mathbb{P} W_1(t)$ (8)

, and for any $a > 0$ let $\tau_a$ be a stopping time defined by

$$\tau_a = \inf \{ t \geq 0 | X_3(t) > a \}. \quad (9)$$

$X_3$ and $\tau_a$ are interpreted as follows. For any $t \in [0, T]$, $\mathcal{G}_t$ shows information on profit/loss of the firm or the business condition at time $t$, and the firm easily can get $\mathcal{G}_t$ with low cost. However it is hard, or it costs too much to measure GHG emissions at every $t \in [0, T]$. Then at every time $t \in [0, T)$, the firm use the business information $\mathcal{G}_t$ that can be related to the volume $X_2$ for the purpose of predicting how much the GHG emissions would be in $[0, T]$. Given a threshold $a$, $\{\tau_a \leq T\}$ is a sign that $X_2$ might be large. Therefore, at time 0, the firm decides the level $a$ such that the sign works for maximizing expected profit. Moreover we will define a stochastic process $X_4$ as follows,

$$\begin{cases} dX_4(t) = u(t, X_4(t))dt + v(t, X_4(t))dW_3(t) \\
X_4(0) = p \end{cases}, \quad (10)$$

where $u$ and $v$ satisfy some conditions. Now we will note modelling a management strategy, where the firm may purchase GHG emission permits additionally depending on a prediction of GHG emissions during $[0, T]$.

maximize \[ E\left[ X_1(T) - K(e^{\tau_T} - 1) - \alpha \max(X_2 - (b(y) + z_1 + c), 0)1_{\{\tau_a > T\}} \right. \]

$$\left. - \left\{ \alpha \max(X_2 - (b(y) + z_1 + z_2 + c), 0) + X_4(\tau_a) \zeta_2(e^{\tau(T-\tau_a)} - 1) \right\} 1_{\{\tau_a \leq T\}} \right], \quad (11)$$

subject to $\quad K = x + y + p z_1, \ x \geq 0, \ y \geq 0, \ a \geq 0,$

where $1_A$ is the indicator function of $A \in \mathcal{F}$. From the view point of management, we explain this optimization problem. At time 0, the firm decides a combination of the business investment
$x$, the emission reduction project investment $y$, and tradable emission permits $z_1$, moreover, the threshold $a$, and additional purchase of tradable emission permits $z_2$ in case that the event $\{\tau_a \leq T\}$ would happen.

Next, in our analysis, price of emissions permits are given at time 0. However, this price has important role of firm’s investment for abatement. So, in relation to this point, two themes which seem to be required to consider remain. One is to model and examine a mechanism that determines the TEPs price. The other is to inquire into change of the firm’s investment portfolio when the TEPs price would change.

Finally, comparing performance or incentives of other different policy instruments such as taxes to abate GHG may also required. For example, Milliman and Prince (1989) compares the incentives provided by five policy instruments - performance standards, emissions subsidies, emissions taxes, and issued and auctioned tradable permits - in the firm-level analysis. The results suggest that the instrument rankings are not invariant. However, they argue that auctioned permits and taxes are the most effective policy instruments to promote new abatement technologies and emissions control. Jung, Krutilla and Boyd(1996) extend Milliman and Prince’ comparative approach from the firm to the industry level. They suggest that the policy instruments can be clearly differentiated in terms of their incentive effects on the development and adoption of new abatement technologies in heterogeneous industries, and show that auction permits provide the most incentive, and issued permits the least. Because our analysis are simple, so considering such as effects of investment decision with environmental tax would be able to do.

There are many points to research further. These will be examined in our another papers.

References


